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RESEARCH DEPARTMENT

REPORT

# A computer program for calculating sky-wave field strengths at medium frequencies

No. 1970/8

Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION

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# A COMPUTER PROGRAM FOR CALCULATING SKY-WAVE FIELD STRENGTHS AT MEDIUM FREQUENCIES


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It uses SI units in accordance with B.S. document PD 5686.

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# A COMPUTER PROGRAM FOR CALCULATING SKY-WAVE FIELD STRENGTHS AT MEDIUM FREQUENCIES

SUMMARY

*A computer program has been developed to calculate sky-wave field strengths at medium frequencies, assuming an idealized horizontally-stratified ionosphere. Ionospheric data and ground constants at the terminals are specified and the losses are computed using a ray-tracing technique. The program will calculate theoretical losses over single hop or two hop paths for ranges between about 100 km and 3,000 km anywhere over the Earth's surface.*

## 1. INTRODUCTION

At medium frequencies (m.f.), radio waves may be propagated between a transmitter and receiver by ground waves or by sky waves. The ground wave, which propagates along and near the surface of the Earth, can be received up to about 500 km from the transmitter. The sky wave, which is reflected from the ionosphere, propagates at night and can be received at much greater distances.

The ionosphere contains free electrons whose density is dependent on the height, time of day, season and sunspot cycle. The reflection of the sky wave is caused by re-radiation by the electrons, which are set into coherent motion by the incident wave. The wave also suffers absorption due to collisions between the free electrons and neutral gas molecules. Both effects may be described by assigning a complex refractive index to the medium, whose value depends on the direction of propagation.

During the day the electron density is so large in the region where the collision frequency is large that the sky wave is almost totally absorbed at m.f. However at night the electron density decreases and only small losses occur. Three regions of reception may then be distinguished. Near the transmitter only the ground wave can be received. At intermediate distances both ground and sky-waves can be received and interference between the two waves causes fading. In the furthest area the sky wave predominates. Since the sky wave can be reflected at the ground and again by the ionosphere, propagation by two or more hops to distances exceeding 3000 km is possible. In these circumstances interference with other co-channel stations is inevitable, and it is highly desirable to be able to predict the field strength of interfering transmitters.

The existing method of field-strength prediction is based on propagation curves published by the EBU and adopted by the CCIR.[1] These propagation curves were obtained from many measurements throughout Europe but they suffer from the disadvantages of being too general and of not including all possible losses.

The computer program described here may be used to calculate the sky-wave field strength which would be observed at m.f. if the wave propagated through a uniform horizontally-stratified ionosphere. Calculations of this nature may be used as a starting point for a study of the effect of the real ionosphere on sky-wave field strength and may eventually form the basis of an improved m.f. field-strength prediction method.

When a wave enters the ionosphere it may, for convenience be resolved into ordinary and extraordinary waves. Each wave is considered separately in the method described here, but if two or more waves are subsequently found to be of comparable strength at the receiver, their power densities are added.

A typical sky-wave path is shown in Fig. 1. The total loss along the transmission path comprises the following basic losses:

(i) Ground loss at the transmitter, due to inefficient reflection of a wave from the ground near the transmitter.[2]

(ii) Polarization coupling loss between the transmitting aerial and the wave entering the ionosphere.[3] This describes the fraction of the incident energy contained in the wave under consideration.

(iii) Ionospheric absorption loss.

(iv) Geometric loss, due to the curvature of the Earth and ionosphere.

(v) Polarization coupling loss and ground reflection loss between hops, if the path takes two or more hops.

(vi) Polarization coupling loss between the downcoming wave and the receiving aerial.

(vii) Ground loss at the receiver.

The methods used for computing these losses are described in the sections which follow. The main part of the program is the ray tracing routine which, using ray optics, plots the path of a ray through the ionosphere and calculates the absorption loss due to electron collisions. The procedure, described in an earlier Research Department Report,[4] involves the use of three terms of an infinite Taylor's series to calculate the co-ordinates of each point on the ray path from the co-ordinates of the previous point.

## 2. GROUND LOSS AT THE TRANSMITTER

There are two contributions to the wave leaving the transmitter site, the direct wave from the radiator and the wave reflected at the ground. The fields of these two waves add vectorially to give the resultant wave leaving the site.

The ground reflection coefficient, for vertically polarized waves, is obtained from the formula

$$\rho_v = \frac{K_r \sin\alpha - (K_r - \cos^2\alpha)^{1/2}}{K_r \sin\alpha + (K_r - \cos^2\alpha)^{1/2}} \qquad (1)$$

where $\alpha$ = angle between the horizontal and the direction of propagation at the Earth's surface

$K_r$ = $(\epsilon_r - jx)$, the complex relative permittivity of the ground

$\epsilon_r$ = dielectric constant of the ground

$x$ = $1{\cdot}8 \times 10^4\ \sigma/f_{MHz}$

$\sigma$ = conductivity, S/m

$f_{MHz}$ = frequency, MHz

As the conductivity of the ground is not infinite the reflected wave will be attenuated. Provided the ground is uniform for many wavelengths in the direction of propagation,[2] the reduction of the resultant wave (ground loss) is given by*

$$20 \log_{10}|1 + \rho_v| - 6\ \text{dB} \qquad (2)$$

## 3. POLARIZATION COUPLING LOSS AT THE TRANSMITTER

The vertically-polarized wave travels undisturbed until it enters the ionosphere, where it may be resolved into two elliptically polarized waves which may be considered independently. These two waves, called the ordinary and extraordinary, follow different paths and incur different ionospheric losses.

The incident energy is shared between the two waves in a ratio which depends upon the angle between the direction of the wave and the Earth's magnetic field vector. The ratio $F_o$ between the power density of the ordinary wave and the incident power density is termed the polarization coupling loss for this wave and is given by[3]

$$F_o = \frac{\cos^2\psi + M^2 \sin^2\psi}{1 + M^2} \qquad (3)$$

where $M$ is the axial ratio of the polarization ellipse and $\psi$ is the angle between the major axis of the ellipse and the vertical, measured anticlockwise looking in the direction of propagation. Formulae for $M$ and $\psi$ are given in Appendix I by Equations (17) and (19) and a full discussion of this effect may be found in Reference 3.

The polarization coupling loss $F_x$ for the extraordinary wave is equal to $1 - F_o$. Expressed in decibels, the polarization coupling losses are $10 \log_{10} F_o$ and $10 \log_{10} F_x$ for the ordinary and extraordinary waves respectively.

## 4. RAY TRACING AND IONOSPHERIC LOSS

When the wave enters the ionosphere, the angle $\phi_1$ between the direction of propagation and the upwards normal is given by

$$\sin\phi_1 = \frac{R\cos\alpha}{R + h} \qquad (4)$$

where $h$ is the height of the base of the ionosphere and $R$ is the radius of the Earth. Ray-tracing is initiated at this point.

The method used for ray tracing is discussed fully in Reference 4. Briefly, the path of the ray is extended by using the first three terms of an infinite Taylor's series representing the ray trajectory to determine the co-ordinates of each new point on the path in terms of the refractive index, the refractive index gradient and the co-ordinates of the previous point. The iterative relationship gives the radial distance $p_{k+1}$ from the centre of the Earth of point $(k + 1)$, in terms of the distance $p_k$ for point $k$, as shown in Fig. 1. The iteration formula is

$$p_{k+1} \simeq p_k + p_k\delta\left[p_k^2\mu_k^2 \sec^2\alpha - 1\right]^{1/2} + \frac{p_k\delta^2}{2}\left\{\left[2 + \frac{p_k}{\mu_k}\left(\frac{d\mu}{dp}\right)_k\right] p_k^2\mu_k^2 \sec^2\alpha - 1\right\} \qquad (5)$$

where $p_k$ and $p_{k+1}$ are distances from the centre of the Earth, relative to the Earth's radius

$\mu_k$ is the real part of the refractive index at point $k$

$\left(\frac{d\mu}{dp}\right)_k$ is the gradient of $\mu_k$ at point $k$

$\delta$ is a specified incremental angle subtended by the two points at the centre of the Earth

* Equation (2) is exact for isotropic and for short vertical transmitting aerials. Antifading mast radiators are subject to an additional ground loss of about 1 dB due to 'height-gain'; this is discussed in Section 2 of Reference 2.

The size of the incremental angle ($\delta$) taken at each iteration will influence the calculation. If the step size is too large there will be inaccuracy due to the size of the higher-order terms in the Taylor series, which have been neglected, and if it is too small the time taken over the whole calculation will be unnecessarily long. Furthermore, ionospheric data is only specified for kilometer steps in height and a linear interpolation is made at other heights. When the ray passes through a kilometer step it will encounter a discontinuity in refractive index gradient due to the linear method of interpolation and some inaccuracy will be introduced however small the step size is made. Although various methods of determining step size were experimented with, including varying step size along the ray path, the present program works most satisfactorily using a constant step size determined by the take-off angle $\alpha$. In practice the range increment along the ground in kilometers ($\delta$ times the Earth's radius) is made numerically equal to $\tan \phi_1$, which is a function of $\alpha$.

The complex refractive index ($\mu - j\chi$) depends upon the Earth's magnetic field vector, the free electron density and the collision frequency at each point along the ray path. It is calculated from the Appleton-Hartree formula[5] stated below.

$$(\mu - j\chi)^2 = 1 - X\left\{1 - jZ - \frac{Y^2 \sin^2\theta}{2(1-X-jZ)} \pm \left[\frac{Y^4 \sin^4\theta}{4(1-X-jZ)^2} + Y^2\cos^2\theta\right]^{1/2}\right\}^{-1} \qquad (6)$$

where $\mu$ = real part of refractive index

$\chi$ = imaginary part of refractive index

$X$ = $80{\cdot}5N/f^2$ where $N$ is the free electron density in number/cm$^3$ and $f$ is the wave frequency in kHz

$Y$ = gyromagnetic frequency divided by the wave frequency

$\theta$ = angle between geomagnetic field and ray direction, given by Equation (18) in Appendix I

$2\pi Z$ = effective collision frequency divided by wave frequency

($\pm$) = + sign for ordinary wave, − sign for extraordinary wave provided $X<1$*

A grid of values of gyromagnetic frequency, dip angle and magnetic variation over the Earth's surface is stored in the computer. The Earth's magnetic field, which is completely specified by these parameters, does not alter appreciably with time. In order to find magnetic field values from the grid at any point along the path, latitude and longitude are first calculated using spherical trigonometry. Magnetic field values are then derived by interpolation from the grid, which contains values every 15° of longitude and 10° of latitude. Interpolation is first made when the wave enters the ionosphere and the values so derived are used for the next 100 km of the path. Further values of magnetic field data are derived subsequently every 100 km along the path.

* Although the ordinary wave is always reflected below the height where $X = 1$, the extraordinary wave may reach this height before reflection occurs. When this situation arises the sign in Equation (6) must be changed to +; this is done in the program. The sign reverts to − when the extraordinary wave descends below the critical height.

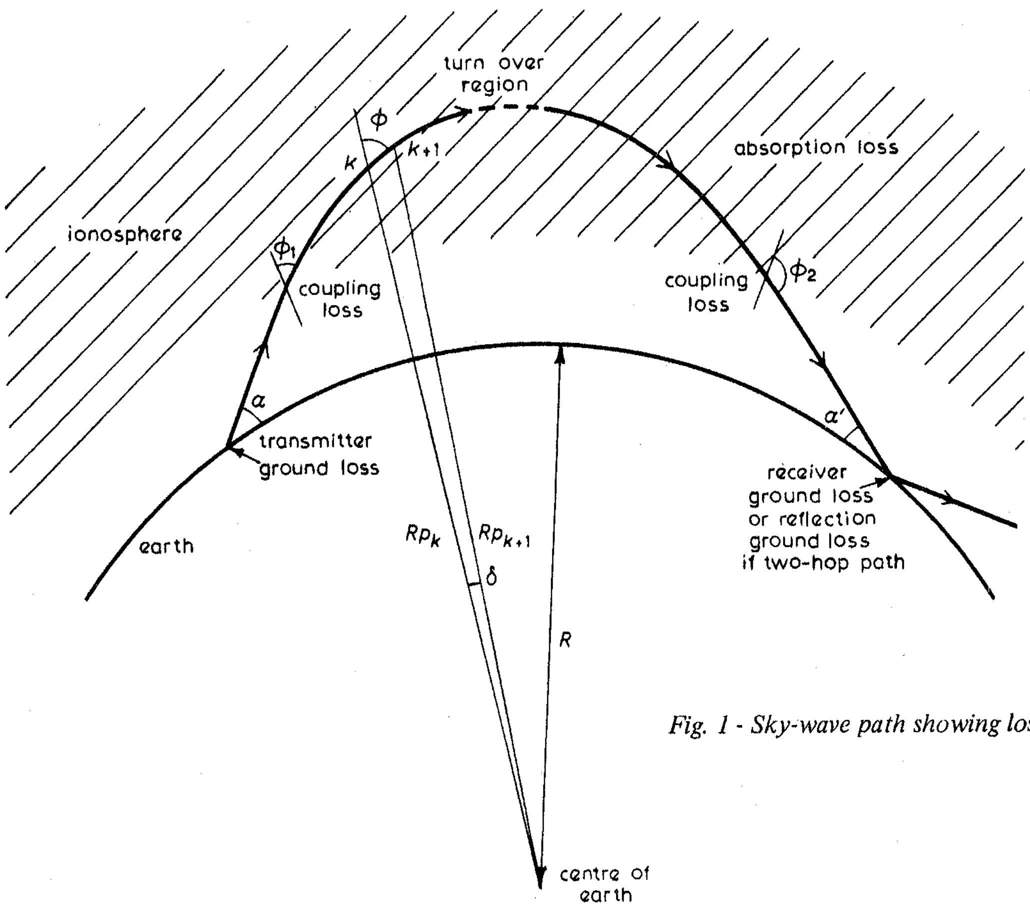


*Fig. 1 - Sky-wave path showing losses*

The absorption in the ionosphere in decibels is found by multiplying the length of each path increment (assumed to be a straight line) by a factor derived from the mean absorption coefficient for the increment. The absorption per km is equal to 54·5 $\chi/\lambda$, where $\lambda$ is the wavelength in km.

On a single-hop path the computer traces the path of the ordinary ray for a specified take-off angle and determines the range and geographical co-ordinates of the point at which it returns to ground. The process is then repeated for the extraordinary ray. To save computer time, the calculation for the extraordinary ray is terminated if its absorption loss exceeds the ordinary ray absorption loss by more than 60 dB. Calculation also ceases if the ray reaches a height in excess of the highest level for which ionospheric data is given; reflection from a higher level might occur if more data were provided.

4.1. Turn-Over Procedure

The iterative process used to find the co-ordinates of each point in the ray path from the previous point may not, in all cases, work correctly at the turnover point. The second term in Equation (5) will pass through zero at turnover but the higher-order terms in the truncated Taylor's series may be appreciable, and their omission may prevent the downward path of the ray from being correctly initiated. Experience has shown that difficulty is most likely to arise if the refractive index at a given height is slightly less on the downward path than it is on the upward path, due to the change in the angle between the ray direction and the Earth's magnetic field. The following method is used to overcome this difficulty.

A comparison is made between the second and third term in the iterative relationship at each step. When the third term exceeds one quarter of the second the ray-tracing calculation is stopped and a turnover routine is called upon. The initial values for the step which stopped the ray-tracing procedure are used at the start of the turnover routine.

Fig. 2 shows the ray path at turnover, the origin of the rectangular co-ordinate system being the point at which the turnover procedure is initiated. The ray direction and refractive index at the origin, denoted by $\phi_o$ and $\mu_o$ respectively, are derived from the ray-tracing procedure. Earth curvature is neglected, since the length of the turnover region is small compared with the radius of the Earth.

The ray direction $\phi$ at a point in the turnover region is related to the refractive index $\mu$ by Snell's Law, which may be written in the form.

$$\operatorname{cosec}\phi = \frac{\mu}{\mu_o}\operatorname{cosec}\phi_o \qquad (7)$$

The ray path may then be defined by the differential equation

$$\frac{dy}{dx} = (\operatorname{cosec}^2\phi - 1)^{1/2} = \left(\frac{\mu^2}{\mu_o^2}\operatorname{cosec}^2\phi_o - 1\right)^{1/2} \qquad (8)$$

where $\mu$ is a function of $y$.

The horizontal distance $\Delta x/2$ covered by the ray in rising to its highest point is therefore

$$\frac{\Delta x}{2} = \int_{y=0}^{y=y_m} dx = \int_o^{y_m} \frac{dy}{\left(\frac{\mu^2}{\mu_o^2}\operatorname{cosec}^2\phi_o - 1\right)^{1/2}} \qquad (9)$$

where $y_m$ is the greatest height reached by the ray.

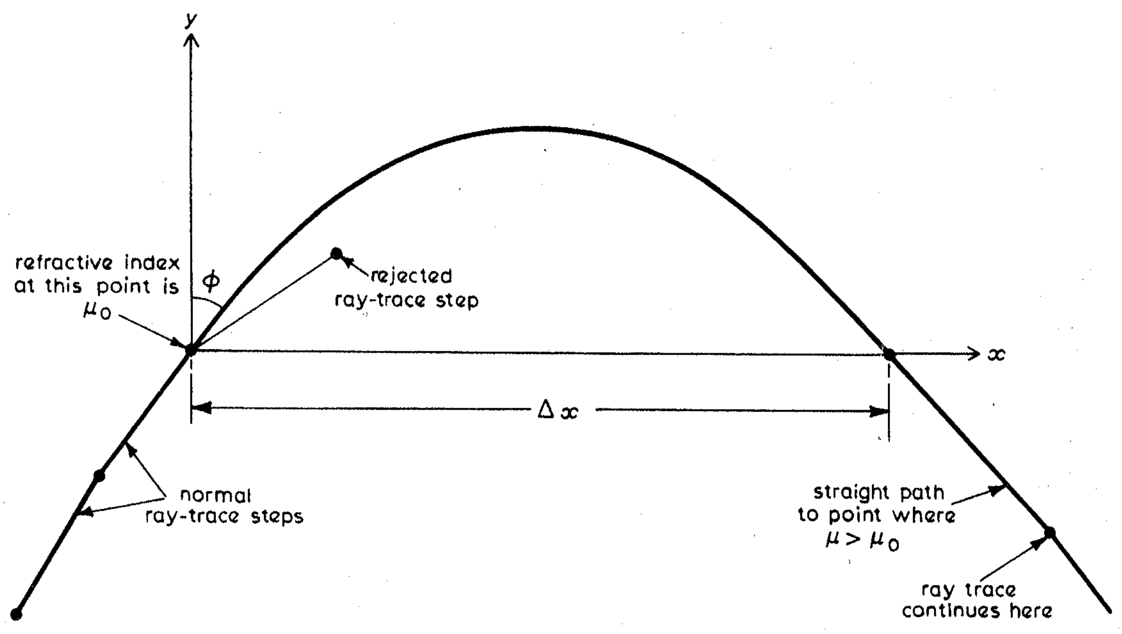


*Fig. 2 - Ray path at turnover*

The integral of Equation (9) may be evaluated if $\mu$ is assumed to be independent of ray direction within the turnover region and to have a vertical gradient $d\mu/dy = G$, which is negative. Since the refractive index at the highest point is equal to $\mu_o \sin\phi_o$, changing the variable in Equation (9) gives

$$\frac{\Delta x}{2} = \frac{1}{G} \int_{\mu_o}^{\mu_o \sin\phi_o} \frac{d\mu}{\left(\frac{\mu^2}{\mu_o^2}\operatorname{cosec}^2\phi_o - 1\right)^{1/2}}$$

$$= \frac{\mu_o \sin\phi_o}{G} \log\left|\frac{\sin\phi_o}{1+\cos\phi_o}\right| \qquad (10)$$

The range increment $\Delta x$ and the associated loss are calculated and the refractive index and refractive index gradient are found for the new angle ($90^\circ + \phi_o$) of the ray. If the refractive index is less than that calculated at the last upward step the neglected terms in the Taylor series will still be too large. In such a case, the ray is extended in a straight line down to the level where the refractive index just exceeds that calculated on the last upward step. Ray tracing then continues as before.

## 5. GEOMETRIC LOSS

This comprises two corrections due to the geometry of the Earth and the ionosphere. The focusing effect of a spherical reflecting layer (sometimes referred to as convergence gain) tends to increase the field strength at the receiver, and an appropriate correction is made. An excess path length correction is also required, because the absolute field strength in the absence of losses is calculated for convenience from the distance measured along the ground, *not* via the ionosphere.* The sum of the two corrections is termed geometric loss; it is positive at short distances and negative at long distances, where convergence gain predominates.

The convergence gain is found from a table based on published curves[6] and stored in the computer, and is subject to an upper limit of 6 dB. The excess path length correction is found from a similar table of values calculated for a range of take-off angles.

## 6. POLARIZATION COUPLING LOSS AND GROUND LOSS AT THE RECEIVER

Both the ordinary and extraordinary waves leaving the ionosphere are, in general, elliptically polarized and further coupling loss arises because m.f. receiving aerials are sensitive mainly to vertical polarization. The loss for the ordinary wave is again given by Equation (3), provided $M$ and $\psi$ are calculated for the new direction of propagation relative to the Earth's magnetic field. The coupling loss for the extraordinary wave is again equal to $1 - F_o$.

* Calculation of the distance along the ground enables the geographical co-ordinates of the point at which the ray returns to Earth to be readily calculated. The distance measured along the ray path could be determined from the ray-tracing procedure if necessary.

Further ground loss occurs when the wave arrives at the receiving aerial, and is calculated as described in Section 2. The angle or arrival may, however, differ slightly from $\alpha$ because the Earth's magnetic field causes slight asymmetry in the path in the ionosphere. The angle of arrival $\alpha'$ is given by

$$\cos\alpha' = \frac{R+h}{R}\sin\phi_2 \qquad (11)$$

where $\phi_2$ is the angle between the ray direction and the upwards normal where the wave leaves the ionosphere.

## 7. FIELD STRENGTH CALCULATION

The field strength at the receiving aerial is calculated by determining the field strength which would be obtained if the transmitting and receiving aerials were situated on perfectly-conducting ground and the ionosphere were a perfect reflector; the actual field strength is then obtained by subtracting the losses discussed in Sections 2 to 6. The transmitting aerial is assumed to be isotropic and to have a field-strength times distance product $(Ed)_o$ of 300 volts. In free space the field strength is therefore $300/d$ mV/m at a distance of $d$ km, but reflection in perfectly-conducting ground at the receiving aerial would double this figure. Thus the basic field strength from which losses are deducted is $600/d$ mV/m; expressed in dB relative to 1 $\mu$V/m this is equal to

$$20 \log_{10}(6 \times 10^5/d) = 115{\cdot}6 - 20 \log_{10} d \qquad (12)$$

where $d$ is the distance between transmitter and receiver in km, measured along the Earth's surface.*

The distance $d$ along the ground is given by

$$d = d_1 + d_2 + nR\delta + d_o \qquad (13)$$

where $d_1$ and $d_2$ are the distances covered before entering and after leaving the ionosphere, $n$ is the number of iterations in the ray-tracing procedure and $d_o$ is the distance traversed in the course of the turnover procedure. It may be shown that

$$d_1 + d_2 = (\phi_2 - \phi_1 - \alpha - \alpha')\frac{\pi R}{180} \qquad (14)$$

where $\phi_1, \phi_2, \alpha$ and $\alpha'$, the angles defining the ray direction, are stated in degrees and shown in Fig. 1.

Although the transmitting aerial is assumed to be isotropic the radiation at angles of less than $10^\circ$ to the

* A correction for the greater distance travelled via the ionosphere is included in the geometrical loss.

horizontal is within 0·1 dB of that which would be obtained if 1 kW were fed to a short vertical aerial. Corrections are required at higher angles, or if the transmitting aerial is not short. In making these corrections, the ground below the aerial is assumed to be perfectly conducting and curves similar to those of Fig. 2 of Reference 1 are used. No further correction need be made for imperfect ground at the transmitter since this correction is included in the ground loss calculation.

## 8. MULTI-HOP PATHS

If there is more than one hop the wave leaving the ionosphere will be reflected by the ground or sea. Some loss will occur at the reflection point and the polarization of the wave will be changed.[3] When the wave enters the ionosphere for the second time its energy will again be shared between ordinary and extraordinary waves.

The wave which leaves the ionosphere after the first ionospheric reflection is the resultant of ordinary and extraordinary waves. It is convenient to consider each component separately; when this is done separate computations must be made for the following four modes:

| First Hop | Second Hop |
|---|---|
| Ordinary | Ordinary |
| Extraordinary | Ordinary |
| Ordinary | Extraordinary |
| Extraordinary | Extraordinary |

Calculations are made of the ratio between the power density of the downcoming ray, and that of the ray re-entering the ionosphere in each case. For the ordinary-ordinary mode the ratio is given by

$$F_{oo} = \frac{|\rho_V(\cos\psi_1 + jM_1\sin\psi_1)(\cos\psi_2 - jM_2\sin\psi_2) + \rho_H(\sin\psi_1 - jM_1\cos\psi_1)(\sin\psi_2 + jM_2\sin\psi_2)|^2}{(1+M_1^2)(1+M_2^2)} \qquad (15)$$

where $M_1$, $M_2$, $\psi_1$ and $\psi_2$ are the axial ratios and orientations of the downcoming and upgoing ordinary-wave polarization ellipses respectively.[3] Ground reflection coefficients for vertical and horizontal polarization are denoted by $\rho_V$ and $\rho_H$ respectively, $\rho_V$ being given by Equation (1) while $\rho_H$ is given by

$$\rho_H = \frac{\sin\alpha - (K_r - \cos^2\alpha)^{1/2}}{\sin\alpha + (K_r - \cos^2\alpha)^{1/2}} \qquad (16)$$

Expressions for the power coupling losses of the other three modes are similar to Equation (15), the differences being that the sign of the appropriate value of $M$ is reversed, and $\cos\psi$ and $\sin\psi$ interchanged, whenever an extraordinary wave is being considered.

Losses for paths consisting of more than two hops may be calculated from the individual losses for two-hop paths. For a three-hop path, for example, all the information required may be obtained by performing separate two-hop computations for hops 1 and 2 and for hops 2 and 3.

If a two-hop path is specified the computer ray-traces for each of the four possible modes and includes the coupling loss between hops among the other losses. The geometric loss is calculated for the first hop only. No further geometric loss arises on the second hop because

(a) the ionospheric focusing on the second hop tends to be cancelled by defocusing at the ground reflection

(b) the ratio between the path lengths along the ground and via the ionosphere is not changed by the second hop.

## 9. LIMITATIONS OF THE METHOD

Since the ray-tracing method makes use of geometrical optics, it is subject to a lower frequency limit which is believed to be at about 0·5 MHz. This figure was derived partly from theoretical considerations and partly from comparisons between ray-tracing and a more exact method, known as the 'full-wave' technique.[7] Since the more exact method requires about 50 times as much computer time as the ray-tracing method, only a limited number of comparisons have been made, although more are envisaged. The frequency limit of 0·5 MHz applies only to 'normal' ionospheric layers; a much higher frequency limit would have to be imposed for electron-density profiles containing sporadic E layers, since these have much steeper refractive-index gradients.

A further limitation arises because the ionosphere is not horizontally stratified but has a cloud-like structure which is continually changing. Consequently any method based on a uniform ionosphere can only provide an estimate of an average field strength, around which there may be considerable variation. In practice diurnal variations may be allowed for by specifying a number of idealized electron-density profiles for various times relative to sunrise and sunset,[8] but short-term, seasonal and solar-cycle variations can only be dealt with on a statistical basis.

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## *APPENDIX I*

### *Formulae for Calculating Wave Polarization*

Wave polarization is defined by the axial ratio and tilt of the polarization ellipse. The tilt for the ordinary wave is given by:

$$\cot\psi = -\sin\phi\ \tan D\ \operatorname{cosec}\gamma - \cos\phi\ \cot\gamma \qquad (17)$$

where

- $\psi$ = rotation from the horizontal, about the ray direction, of the minor axis of the electric-field polarization ellipse. Looking along the ray direction, this rotation is anticlockwise if $\psi$ is positive.
- $\phi$ = angle between the ray direction and the upwards normal ($0<\phi<90^\circ$ on upward path, $90^\circ<\phi<180^\circ$ on downward path)
- $D$ = magnetic dip angle ($-90^\circ \leqslant D \leqslant 90^\circ$, positive to north of magnetic equator)
- $\gamma$ = magnetic bearing of receiver from transmitter, measured east of north.

Before the axial ratio can be calculated the angle $\theta$ between the ray direction and the Earth's magnetic field must be calculated. This angle, which is also required for refractive-index calculations, is given by

$$\cos\theta = \sin\phi\ \cos D\ \cos\gamma - \cos\phi\ \sin D \qquad (18)$$

The axial ratio $M$ of the polarization ellipse is given by

$$M = \tan\frac{\omega}{2} \qquad (19)$$

where

$$\cot\omega = \frac{Y}{2}\sin\theta\ \tan\theta \qquad (-90^\circ \leqslant \omega \leqslant 90^\circ) \qquad (20)$$

$Y$ = gyromagnetic frequency divided by wave frequency.

If the wave frequency is equal to the gyro frequency ($Y$ = 1) the expression for the axial ratio simplifies to $M = \cos\theta$. $M$ lies in the range $-1 \leqslant M \leqslant 1$; a positive value means that the sense of polarization is clockwise looking in the direction of propaction.

The formulae for $M$ and $\psi$ given above are for the ordinary ray. The axial ratio and tilt of the extraordinary ray are equal to $-M$ and $90^\circ - \psi$ respectively.

## *APPENDIX II*

### *Computer Input and Output*

The title of the program is 'MF propagation between specific sites' and it must be used in conjunction with magnetic field data stored either on film or on tape. It is written in Elliott 803 Autocode Mark 3.

An annotated example of the normal input data is shown in Fig. 3. Electron density and collision frequency data are read in at 1 km height intervals, starting from a specified lowest height. Up to 700 electron-density values may be specified but the number need not be as large as this; for example if the profile is terminated at the top of the E layer (as in Fig. 3) less than 100 values will be required. The need for specifying a large number of collision-frequency values is avoided by terminating the data in a zero as soon as the collision frequency is too small to have any significant effect; collision frequencies for all greater heights are then automatically set to zero.

The direction of the receiver, frequency and take-off angle are specified in the form

Initial value : increment : final value

The computer then works through all combinations of the specified values in sequence. One value only of any of these parameters can be specified by making the final value less than the sum of the initial value and increment.

The take-off angle required for a given range must be estimated from charts such as those contained in Reference 6. To calculate field strength at a specific site a number of take-off angles must be chosen so as to straddle the receiver with rays; the field strength at the specified site is then found by interpolation.

If the initial frequency is specified as −1 and the remaining frequency and take-off angle data omitted, the program will automatically compute for frequencies of 0·5, 0·7, 1·0 and 1·5 at take-off angles of 1, 2, 4, 6, 10, 15, 20, 25 and 30 degrees.

The amount of frequency and path data which can be specified for a given ionospheric profile is limited only by the storage space allocated to it. If too much is specified the computer prints 'TOO MUCH DATA' and stops.

Fig. 4 shows the output obtained with the data specified in Fig. 3. It will be seen that the computer prints 'ESCAPE' if the ray reaches the greatest height at which electron density data has been specified without being reflected. The computer will also print 'LARGE' and proceed to the next computation if the extraordinary-wave absorption exceeds the ordinary-wave absorption by more than 60 dB. The output can be stored on film if desired.

A keyboard facility exists which enables the results of each step of the iteration procedure to be printed, as shown in Fig. 5. This permits a detailed study of the progress of the ray through the ionosphere to be made but it slows down the computer. The turn-over step is indicated by an asterisk.

A variation of the program with the title 'ray tracing' also exists. The principal difference is that it uses fixed, specified, magnetic field data rather than world data and its normal output is similar to Fig. 5. Its use enables the progress of a ray to be studied theoretically under controlled conditions.

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ROME-TATSFIELD 845 KHZ
TWO HOURS AFTER SUNSET

62
```

Lowest height at which ionospheric data is specified (km)

```
0  0  0  0  0  0  0  0  .75
1.2  1.5  2  2.6  3.4
4.5  6.1  8.6  12  18
27  40  60  90  130
180  260  380  520  720
960  1250  1550  1950  2400
2900  3500  4000  4700  5300
5900  6500  7000  7400  7800
8100  8300  8400  8500  8500
8400  8300  8100  7900  7700
7400  7100  6800  6400  6000
5800  5600  5400  5200  5000
4800  4600  4400  4200  4000
```

Electron density at 1 km height intervals, starting at the height specified above (electrons per cc)

```
-1
```

Trigger denotes end of electron-density data

```
3700  3150  2650  2300
1900  1620  1400  1240  1080
920  800  700  604  508
444  392  318  266  220
188  153  127  105  86
70  59.2  47.6  39.2  31.8
27.4  23  19.4  16.2  13.8
11.8  10.2  8.2  7  6
5  4.4  3.8  3.2  2.7
2.4  2  1.7  1.5  1.3
1.1  1  .8  .7  .6
.5  .44  .38  .3  .25
.2  .15  .1  .05  0
```

Effective collision frequency divided by $2\pi$ at 1 km height intervals, starting at the height specified above (kHz)

This need not contain as many values as the electron-density data. The last value is denoted by zero.

ONE-HOP DATA

```
41.7  12.6
322.4  1  322.4
.845  1  .845
3.7  1  3.7
1
15  .01
15  .01
```

Latitude (N of Equator) and longitude (E) of transmitter
Direction from transmitter (degrees E of true North)
Frequency (MHz)
Take off angle (degrees to horizontal)
Number of hops
Dielectric constant and conductivity (S/m) at transmitter
Dielectric constant and conductivity (S/m) at receiver

TWO-HOP DATA

```
41.7  12.6
322.4  1  322.4
.845  1  .845
12.8  1  12.8
2
15  .01
10  .003
15  .01
```

Further data for 2-hop propagation as above, but with addition of :–

ground constants at intermediate reflection point

```
-2
```

Trigger denotes end of data

*Fig. 3 - Input data*

```
TRANSMITTER  LAT/LONG =  41.7  /  12.6

BEARING = 322.4  FREQUENCY =  0.845  MHZ
```

ONE – HOP OUTPUT

| | | | |
|---|---|---|---|
| ANGLE = 3.7 | | | |
| MODE | O | X | Ordinary or extraordinary mode |
| TG LOSS | 5.7 | 5.7 | Ground loss at transmitter |
| TX LOSS | 0.6 | 9.2 | Polarization coupling loss at transmitter |
| I1 LOSS | 15.0 | ESCAPE | Ionospheric absorption loss |
| GM LOSS | -4.1 | | Geometric loss |
| RX LOSS | 0.9 | | Polarization coupling loss at receiver |
| RG LOSS | 4.9 | | Ground loss at receiver |
| T LOSS | 22.9 | | Total loss (dB) |
| E FIELD | 29.5 | | Field strength (dB$\mu$) for $(Ed)_0$ = 300 volts |
| RANGE 1 | 1432.1 | | Distance from transmitter (km), latitude and longitude of receiving point |
| LAT | 51.3 | | |
| LONG | 0.0 | | |

```
TRANSMITTER  LAT/LONG =  41.7  /  12.6

BEARING = 322.4  FREQUENCY =  0.845  MHZ
```

TWO – HOP OUTPUT

| | | | | | |
|---|---|---|---|---|---|
| ANGLE = 12.8 | | | | | |
| MODE | O-O | O-X | X-O | X-X | Modes for first and second hops respectively |
| TG LOSS | 1.9 | 1.9 | 1.9 | 1.9 | Ground loss at transmitter |
| TX LOSS | 0.5 | 0.5 | 9.3 | 9.3 | Polarization coupling loss at transmitter |
| I1 LOSS | 15.3 | 15.3 | ESCAPE | ESCAPE | Ionospheric absorption loss on first hop |
| GR LOSS | 21.0 | 4.1 | | | Ground and coupling loss between hops |
| I2 LOSS | 15.5 | ESCAPE | | | Ionospheric absorption loss on second hop |
| GM LOSS | -0.5 | | | | Geometric loss |
| RX LOSS | 1.1 | | | | Polarization coupling loss at receiver |
| RG LOSS | 1.8 | | | | Ground loss at receiver |
| T LOSS | 56.6 | | | | Total loss (dB) |
| E FIELD | -4.2 | | | | Field strength (dB$\mu$) for $(Ed)_0$ = 300 volts |
| RANGE 1 | 726.6 | | | | Distance from transmitter (km), latitude and longitude of Earth reflection point |
| LAT | 46.7 | | | | |
| LONG | 6.8 | | | | |
| RANGE 2 | 1436.5 | | | | Distance from transmitter (km), latitude and longitude of receiving point |
| LAT | 51.3 | | | | |
| LONG | 360.0 | | | | |

*Fig. 4 - Normal output*

All losses are in dBs

| Range (km) | Height (km) | Ionospheric absorption loss (dB) | Refractive index real part | Refractive index imaginary part | cos $\phi$ |
|---|---|---|---|---|---|
| 551.3 | 60.01 | 0.00 | 1.00000 | 0.00000 | 0.14953 |
| 557.9 | 61.03 | 0.00 | 1.00000 | 0.00000 | 0.14953 |
| 564.6 | 62.06 | 0.00 | 1.00000 | 0.00000 | 0.15353 |
| 571.2 | 63.10 | 0.00 | 1.00000 | 0.00000 | 0.15459 |
| 577.8 | 64.14 | 0.00 | 1.00000 | 0.00000 | 0.15565 |
| 584.4 | 65.19 | 0.00 | 1.00000 | 0.00000 | 0.15671 |
| 591.0 | 66.24 | 0.00 | 1.00000 | 0.00000 | 0.15776 |
| 597.6 | 67.31 | 0.00 | 1.00000 | 0.00000 | 0.15882 |
| 604.2 | 68.38 | 0.00 | 1.00000 | 0.00000 | 0.15989 |
| 610.8 | 69.46 | 0.00 | 0.99999 | 0.00001 | 0.16095 |
| 617.5 | 70.54 | 0.02 | 0.99998 | 0.00003 | 0.16195 |
| 624.1 | 71.63 | 0.06 | 0.99996 | 0.00004 | 0.16288 |
| 630.7 | 72.73 | 0.10 | 0.99994 | 0.00005 | 0.16386 |
| 637.3 | 73.84 | 0.16 | 0.99991 | 0.00006 | 0.16478 |
| 643.9 | 74.95 | 0.24 | 0.99987 | 0.00008 | 0.16563 |
| 650.5 | 76.06 | 0.33 | 0.99980 | 0.00010 | 0.16639 |
| 657.1 | 77.18 | 0.45 | 0.99970 | 0.00013 | 0.16701 |
| 663.7 | 78.30 | 0.61 | 0.99954 | 0.00017 | 0.16737 |
| 670.4 | 79.43 | 0.81 | 0.99927 | 0.00021 | 0.16723 |
| 677.0 | 80.54 | 1.07 | 0.99882 | 0.00028 | 0.16640 |
| 683.6 | 81.64 | 1.40 | 0.99814 | 0.00036 | 0.16423 |
| 690.2 | 82.72 | 1.83 | 0.99711 | 0.00046 | 0.16045 |
| 696.8 | 83.75 | 2.37 | 0.99559 | 0.00058 | 0.15385 |
| 703.4 | 84.70 | 3.03 | 0.99366 | 0.00069 | 0.14313 |
| 710.0 | 85.56 | 3.79 | 0.99153 | 0.00077 | 0.12856 |
| 716.6 | 86.30 | 4.63 | 0.98912 | 0.00086 | 0.11090 |
| 723.3 | 86.87 | 5.57 | 0.98678 | 0.00095 | 0.08631 |
| 729.9 | 87.24 | 6.58 | 0.98491 | 0.00101 | 0.05557 |
| 751.4 | 87.24 | 9.93 | 0.98578 | 0.00094 | -0.05557 |
| 758.0 | 86.78 | 10.85 | 0.98821 | 0.00085 | -0.06947 |
| 764.6 | 86.08 | 11.66 | 0.99118 | 0.00071 | -0.10513 |
| 771.3 | 85.21 | 12.34 | 0.99350 | 0.00062 | -0.13037 |
| 777.9 | 84.26 | 12.93 | 0.99543 | 0.00052 | -0.14272 |
| 784.5 | 83.23 | 13.42 | 0.99700 | 0.00042 | -0.15332 |
| 791.1 | 82.15 | 13.81 | 0.99810 | 0.00032 | -0.16095 |
| 797.7 | 81.05 | 14.10 | 0.99881 | 0.00025 | -0.16510 |
| 804.3 | 79.93 | 14.33 | 0.99926 | 0.00019 | -0.16705 |
| 810.9 | 78.80 | 14.50 | 0.99954 | 0.00014 | -0.16789 |
| 817.5 | 77.67 | 14.64 | 0.99970 | 0.00012 | -0.16797 |
| 824.2 | 76.55 | 14.75 | 0.99980 | 0.00009 | -0.16750 |
| 830.8 | 75.43 | 14.83 | 0.99987 | 0.00007 | -0.16691 |
| 837.4 | 74.32 | 14.90 | 0.99991 | 0.00006 | -0.16610 |
| 844.0 | 73.21 | 14.95 | 0.99994 | 0.00004 | -0.16524 |
| 850.6 | 72.11 | 14.99 | 0.99996 | 0.00003 | -0.16432 |
| 857.2 | 71.01 | 15.02 | 0.99997 | 0.00003 | -0.16336 |
| 863.8 | 69.92 | 15.04 | 0.99999 | 0.00001 | -0.16235 |
| 870.4 | 68.84 | 15.05 | 1.00000 | 0.00000 | -0.16137 |
| 877.1 | 67.77 | 15.05 | 1.00000 | 0.00000 | -0.16038 |
| 883.7 | 66.70 | 15.05 | 1.00000 | 0.00000 | -0.15928 |
| 890.3 | 65.64 | 15.05 | 1.00000 | 0.00000 | -0.15823 |
| 896.9 | 64.59 | 15.05 | 1.00000 | 0.00000 | -0.15716 |
| 903.5 | 63.54 | 15.05 | 1.00000 | 0.00000 | -0.15611 |
| 910.1 | 62.51 | 15.05 | 1.00000 | 0.00000 | -0.15505 |
| 916.7 | 61.48 | 15.05 | 1.00000 | 0.00000 | -0.15505 |
| 923.3 | 60.45 | 15.05 | 1.00000 | 0.00000 | -0.15505 |
| 930.0 | 59.44 | 15.05 | 1.00000 | 0.00000 | -0.15505 |

*Fig. 5 - Step-by-step output*